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THE TWO DIMENSIONAL INFLOW CONDITIONS FOR A SUPERSONIC COMPRESSOR WITH CURVED BLADES

PHILIP LEVINE

AERONAUTICAL RESEARCH LABORATORY

MAY 1956

WRIGHT AIR DEVELOPMENT CENTER

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WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
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FOREWORD

WADC TR 55-387

ABSTRACT

Results are presented on an analytical study of the flow field existing upstream of a blade row, where the axial flow is subsonic and the relative flow is supersonic. The flow model used as a basis for the calculations, assumes isentropic flow, and considers the case where the suction surface is a circular arc in the entrance region.

The results clearly show the unique dependence of the flow through a blade row upon the geometry of the entrance region. Using the results, the complete flow field in the entrance region and upstream of the blade row can be easily constructed.

PUBLICATION REVIEW

This report has been reviewed and is approved.

Aldro Lingard

ALDRO LINGARD
Colonel, USAF
Chief, Aeronautical Research Laboratory
Directorate of Research

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LIST OF SYMBOLS

A - area normal to the flow direction

A^* - critical area normal to the flow direction

e - point on the suction surface marking the end of the entrance region

L - leading edge

M - Mach number

R - radius of curvature of the suction surface in the entrance region

S - blade spacing

∞ - point on the suction surface where the Mach number and direction of the flow are the same as the relative undisturbed flow quantities.

β_t - angle formed by the tangent to the suction surface at the leading edge with the plane of the blade row.

β_∞ - angle formed between the direction of the relative undisturbed flow and the plane of the blade row.

θ - angular measurement along the suction surface taken from the leading edge

μ - Mach angle

SUBSCRIPTS

L - conditions at the leading edge

∞ - conditions of the undisturbed flow and at the point (∞) on the suction surface

e - conditions at the point (e) on the suction surface

INTRODUCTION

A distinguishing characteristic of supersonic compressors, having subsonic axial flow, is the unique dependence of the corrected weight flow on the corrected rotor speed. This characteristic has been discussed in Reference 1, where it is indicated that the axial velocity is controlled by waves generated at the entrance region of the blades.

The complete construction of the inlet flow field for blading with straight suction surfaces was shown (Ref. 1) to be relatively simple, as compared to that for curved blades. Since there is considerable interest in curved blades for supersonic compressors, it was deemed worthwhile to investigate the possibility of finding a direct method for calculating the flow field in the entrance region of curved blades. As a result of this investigation, a relatively simple solution was found, based on the premise that the suction surface in the entrance region is a circular arc and on the assumptions that the flow about the suction surface in the entrance region isentropic.

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PART I - DESCRIPTION OF THE FLOW FIELD

THE FLOW MODEL

The problem under consideration, is the determination of the flow field established upstream of an annular cascade or rotor under the conditions that the undisturbed flow relative to the blades is supersonic and that the axial flow is subsonic and steady.

It is assumed that the annular cascade or rotor is nearly two dimensional (i. e., high hub ratio) and can be represented by an equivalent infinite two-dimensional cascade. Thus, hereafter, (for convenience) an annular cascade, rotor or infinite cascade are referred to simply as a blade row.

The geometry of a typical blade row and the associated notation to which reference will be made in the text is shown in Figure 1. The axis of the blade row is normal to the plane of the blade row. The geometry of the blade row is fixed by the blade spacing S, the radius of curvature R, of the portion of the suction surface which lies in the entrance region, (entrance region will be defined later) and the angle (β_t) made by the tangent to the suction surface with the plane of the blade row at the leading edge. The suction surface, as used here, is defined as that surface whose tangent at the leading edge forms the smallest angle with the plane of the blade row. Points on the suction surface will be located by the angle (θ), measured from the leading edge.

The flow upstream, free of disturbances arising as a result of the blade row being in the flow is referred to as the undisturbed flow. The undisturbed flow is taken to be uniform (i. e., one-dimensional).

LIMITING CASE: SONIC AXIAL FLOW

To start the study of a flow into a blade row, it may be well to consider the relatively simple limiting case, where the axial Mach number of the undisturbed flow is supersonic but arbitrarily close to unity. For this special case, the flow immediately ahead of the blade row is disturbance free, due to the fundamental character of the sonic flow which does not permit disturbances to travel upstream. Therefore, for steady flow, the wave system generated by the blade row lies inside the blade passages.

For this case, the angle (β_∞) that the relative inlet flow makes with the plane of the blade row as shown in Figure 2, is all that is needed to describe the flow field ahead of the blade row. Three possibilities exist for the value of (β_∞); (1) $\beta_\infty = \beta_t$; (2) $\beta_\infty < \beta_t$; (3) $\beta_\infty > \beta_t$. The first possibility sets the relative inlet Mach number as tangent to the suction surface at the leading edge, forming a Mach wave with Mach angle (μ_∞), as shown in Figure 2 (a), where one observes that

$$M_\infty = \frac{1}{\sin\beta_t}$$

but by definition,

$$M_\infty = \frac{1}{\sin\mu_\infty}$$

Therefore $\mu_\infty = \beta_t$, and the Mach wave formed at the leading edge of one blade, strikes the leading edge of the adjacent blade, so that no contradictions exist and steady flow is possible. The second possibility, where $\beta_\infty < \beta_t$ is shown in Figure 2(b), where the relative inlet Mach number strikes the leading edge of the suction surface with a positive incidence angle. Here, an expansion starts at the leading edge, and one observes from Figure 2(a), that

$$M_\infty = \frac{1}{\sin\beta_\infty} = \frac{1}{\sin\mu_\infty}$$

Thus, the first characteristic of the expansion wave formed at the leading edge just strikes the leading edge of the adjacent blade, so that no contradictions exist and steady flow is possible. The third possibility where $\beta_\infty > \beta_t$ is shown in Figure 2(c) where the relative inlet flow strikes the leading edge of the suction surface with a negative incidence angle. Here, an oblique shock is formed at the leading edge. If a Mach wave were formed, then by a similar argument as above, it would strike the leading edge of the adjacent blade. But the wave angle of an oblique shock is greater than the corresponding Mach angle, so that the shock would extend upstream of the leading edge, violating the condition of steady sonic flow. Hence the third possibility cannot exist. Thus, when the inlet angle is increased such that $\beta_\infty > \beta_t$, transient waves move upstream to reduce the mass flow. The resultant axial Mach number must be subsonic for if it were supersonic the incidence angle would be further decreased which would only aggravate the incompatibility that established the transient waves in the first place.

The result of the above analysis indicates that the smallest value the relative inlet Mach number may have for a given blade row geometry and still support steady sonic axial flow occurs when $\beta_\infty = \beta_t$ and $M_\infty = \dfrac{1}{\sin\beta_t}$. Reducing the relative inlet Mach number below $\dfrac{1}{\sin\beta_t}$ causes a readjustment of the flow resulting in a subsonic axial Mach number.

GENERAL CASE: SUBSONIC AXIAL FLOW:

Continuing along the line of study initiated above, the construction of the flow field resulting when steady subsonic axial flow is established is then the next step.

It was shown above, that when the relative inlet Mach number is reduced below $\dfrac{1}{\sin\beta_t}$, an oblique shock (assumed to be attached for the present argument) is formed at the leading edge, as shown in Figure 3(a). From Figure 3(a), one observes, that an expansion fan is generated along the suction surface immediately behind the shock. In Figure 3(b) the effect of the shock and expansion, emanating from one blade, on the streamline which just strikes the leading edge of the adjacent blade is shown. The flow along the characteristic (e-L) has been turned through the angle (θ_e) from the direction at the leading edge by the expansion wave, so that the incident angle of the flow on the leading edge of the adjacent blade is the angle (θ_e), giving rise to another oblique shock on the adjacent blade.

Continuing in this manner, one finds that an oblique shock followed by an expansion exists for each blade, as shown in Figure 4. One observes from Figure 4, that expansion waves strike the shock wave from both sides, forming a rather complex wave. A small part of an expansion wave striking a shock is reflected; however in these considerations, these reflections are neglected, introducing a small error when strong shocks are present.

To establish steady flow for the desired value of the undisturbed relative Mach number, M_∞ , the net strength of the shocks and expansion waves extending upstream must cancel, for if a disturbance of finite strength travels upstream, the mass flow will change violating the condition of steady flow. For a uniform row of blades, the portion of the wave system arising from each blade will be the same, so that the net strength of the disturbances arising from each blade which travel

upstream, must be zero. Now, the point (e) on the suction surface is defined such that the characteristic generated there is just intercepted by the leading edge of the adjacent blade. Hence, the portion of the expansion fan downstream of the point (e) is enclosed between the blades, along with the wave system arising from the pressure surface of the blades. Consequently, in accordance with Figure 4, only that portion of the blading and hence of the expansion fan in the region bounded by the points (LeL) (hereafter designated as the entrance region) has any influence on the inlet flow pattern. This of course excludes special considerations such as starting due to a throat in the blade passage, and presumes that the leading edge is a point so that regardless of the location of the point (e), the characteristic emanating there will always be intercepted at the exact leading edge (point (L) in Figure 4) of the adjacent blade.

From the above analysis of the wave system, it is apparent that only waves of one family, (generated by the suction surfaces) exist upstream of the characteristic (e-L). Thus, the flow in the entrance region is a Prandtl-Meyer expansion. To relate the flow quantities along the characteristic (e-L) to the condition at infinity, one can now utilize continuity and the relation for a Prandtl-Meyer expansion.

Prescribing the location of the point (e) relative to (L), and the slope of the suction surface at (e), determines the Mach number of the flow and the flow direction there. Consequently, the mass flow into the blade channel is fixed, once point (e) and the slope of the suction surface at that point are defined. By continuity, and the relation for a Prandtl-Meyer expansion, one has two equations at his disposal to relate the Mach number and direction of the flow at infinity (far upstream) to the flow quantities along the characteristic (e-L).

The approach could be reversed, so that one could specify the flow conditions at infinity, and then determine the location of (e) relative to (L), and the slope of the suction surface at (e).

It should be emphasized, that no special shape has been attributed to the suction surface as yet, but merely the relative location of the points (e) and (L) and the slope of the suction surface at (e).

For the case of a rotor, it is now apparent, that changing the wheel speed, changes the direction of the flow relative to the blades, so that transient waves are sent upstream to adjust the axial Mach number (hence

mass flow) in accordance with the steady flow and continuity requirements discussed above. The ability of transient waves to travel upstream stems from the fact that the axial Mach number is subsonic. Thus, for a fixed geometry, each rotor speed will correspond to a particular mass flow.

PART II - ANALYTICAL METHOD

The conditions which connect the flow field far upstream of the cascade to the flow field along the characteristic (e-L) can be formulated as follows:

1. Continuity- The mass flow entering a blade passage (i.e., crossing e-L) equals the mass flow passing through a cross-section of length S at infinity.

2. The waves originating from the suction surface in the entrance region are of only one family, (i.e., Prandtl-Meyer expansion).

The flow conditions along the characteristic (e-L) are related to the upstream undisturbed flow, by the continuity equation and the geometry of Figure 6, such that,

$$\frac{S}{A_e} \sin(\beta_t + \theta_\infty) = \frac{M_e}{M_\infty} \left\{ \frac{1 + [(\gamma-1)/2]M_\infty^2}{1 + [(\gamma-1)/2]M_e^2} \right\}^{\frac{\gamma+1}{2(\gamma-1)}} \tag{1}$$

where
$$\frac{M_e}{M_\infty} \left\{ \frac{1 + [(\gamma-1)/2]M_\infty^2}{1 + [(\gamma-1)/2]M_e^2} \right\}^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{A_\infty}{A^*} \frac{A^*}{A_e}$$

The angles (θ_∞) and (θ_e) are connected by the Prandtl-Meyer relationship,

$$\theta_\infty = \theta_e + \nu_\infty - \nu_e$$

where   (2)

$$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}} \; \text{Tan}^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} \sqrt{M^2-1}} \; - \; \text{Tan}^{-1} \sqrt{M^2-1}$$

Consequently, (θ_∞) may be eliminated from equation 1, so that

$$\frac{S}{A_e} \sin(\beta_t + \theta_e + \nu_\infty - \nu_e) = \frac{M_e}{M_\infty} \left\{ \frac{1+[(\gamma-1)/2]M_\infty^2}{1+[(\gamma-1)/2]M_e^2} \right\}^{\frac{\gamma+1}{2(\gamma-1)}}$$

(3)

Now equation 3, represents a general relationship between the Mach number (M_e) along the characteristic (e-L) and the undisturbed relative flow Mach number (M_∞), and is not restricted to any particular shape of the suction surface between the points (e) and (L). By specifying the location of the point (e) and the slope of the suction surface there (which in equation 3 amounts to specifying A_e and θ_e) then the Mach numbers (M_e) and (M_∞) are directly related by the blade geometry parameters (S, β_t).

SOLUTION FOR A CONVEX SUCTION SURFACE

Specifying the shape of the suction surface from (L) to (e), permits one to relate (S) and (A_e) through the angles $(\beta_t, \theta_e, \mu_e)$. For the special case of a convex circular are from (L) to (e), the following relationships can be derived from Figure 6.

$$\frac{A_e}{\sin \mu_e} \sin(\mu_e - \beta_t - \theta_e) = R[\cos \beta_t - \cos(\beta_t + \theta_e)]$$ (4)

$$\frac{A_e}{\sin \mu_e} \cos (\mu_e - \beta_t - \theta_e) = S - R \left[\sin (\beta_t + \theta_e) - \sin \beta_t \right] \tag{5}$$

Combining equations 4 and 5 and solving for $\dfrac{A_e}{S} = f(\mu_e \beta_t, \theta_e)$ and introducing $\sin \mu_e = \dfrac{1}{M_e}$ yields,

$$\frac{A_e}{S} = \frac{\cos \beta_t - \cos (\beta_t + \theta_e)}{\sin \theta_e - (1 - \cos \theta_e)\sqrt{M_e^2 - 1}} \tag{6}$$

Eliminating $\left(\dfrac{A_e}{S} \right)$ by combining equations 3 and 6 yields

$$\frac{(1 + [(\gamma-1)/2]M_\infty^2)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_\infty \cos \nu_\infty \left[\mathrm{Tan}(\beta_t + \theta_e - \nu_e) + \mathrm{Tan}\, \nu_\infty \right]} =$$

$$\frac{(1 + [(\gamma-1)/2]M_e^2)^{\frac{\gamma+1}{2(\gamma-1)}} \cos \beta_t + \theta_e - \nu_e)[\sin \theta_e - (1 - \cos \theta_e)\sqrt{M_e^2 - 1}]}{M_e[\cos \beta_t - \cos (\beta_t + \theta_e)]} \tag{7}$$

Equation 7 relates $M_\infty = f_\infty (\beta_t, M_e, \theta_e)$, with the terms on the right hand side known as a result of specifying the location of (e) relative to (L) and the slope at (e).

It is of practical interest, to relate $M_\infty = f_s (S/R, \beta_t, \theta_e)$, which can be done by combining equations 4 and 5, eliminating (A_e) and solving for (M_e),

$$M_e = \sqrt{1 + \left[\frac{S/R \cos (\beta_t + \theta_e) - \sin \theta_e}{S/R \sin (\beta_t + \theta_e) - 1 + \cos \theta_e} \right]^2} \tag{8}$$

Substituting equation (8) into (7) yields M_∞ (S/R, β_t, θ_e). The use of (M_∞) as the dependent variable in several relationships above, may be taken as part of the result, that (M_∞) is dependent on the geometry of the blade row, and the location of the point (e). However, it is more convenient to display the calculated values of the above relationships in charts with (M_∞) as a curve parameter. These charts are located at the end of the text.

SOLUTION FOR A CONCAVE SUCTION SURFACE

The solution for the concave suction surface is identical with the convex solution, as a reversed Prandtl-Meyer flow occurs along the surface (L) to (e) as shown in Figure 7. Care must be taken to observe the sign on the angles (θ_e) and (θ_∞). For the convex solution these were taken as positive angles, while for the concave solution they should be taken as negative. This follows from the reversed nature of the flow.

Thus, equations 1 to 8 are valid for this case provided that (θ_e, θ_∞) are replaced by ($-\theta_e$) and ($-\theta_\infty$) respectively wherever they occur.

PART III - DISCUSSION OF RESULTS AND CONCLUSIONS

The calculated results are shown in the charts for values of $15° \leq \beta_t \leq 40°$ (taken in $5°$ increments) located at the rear of the text. The abcissa of each chart, is the ratio (S/R), where (R) is the radius of the suction surface in the entrance region. Thus, fixing (β_t) and (S/R) satisfies all the geometrical conditions. The ordinate is (M_e), which is the local Mach number occurring along the expansion ray (e-L), (See Fig. 6). The lines labeled (θ_e) locate the point (e) on the blade in degrees measured from the leading edge. The lines labeled (M_∞) are values of the undisturbed relative Mach number. Any point on the chart satisfies both families of curves.

In addition to what is presented in the charts, a large amount of information can be readily found by using the charts in combination with Prandtl-Meyer tables.

DISCUSSION OF RESULTS

Practically, it is of interest to determine the variation of the Mach number from (L) to (e). It may be noted at this point, that ⬛⬛⬛⬛⬛⬛⬛⬛,

the Mach number and flow along the characteristic (e-L) ~~~~~~~~~~ *cannot* ~~~~~~ be the undisturbed value (M_∞). However, such a condition leads to a contradiction in applying the continuity relationship. This may be seen from Figure 5, where one observes that the streamlines of the undisturbed relative inlet flow, containing the flow which must pass through each blade passage are spaced at a distance (S). By continuity, if the flow along the ray (e-L) has the same Mach number and direction as the relative undisturbed Mach number and direction, then the stream-line spacing must also be (S). That this is not so, is apparent from Figure 5 where the spacing of the streamlines (Se). along the character-istic (e-L) is clearly shorter than the distance (S). Hence continuity cannot be satisfied and the undisturbed flow conditions cannot exist along (e-L). To satisfy the condition that the net strength of the wave system extending upstream is zero, the expansions striking each shock must be of just sufficient strength to eventually cancel the shock. From Figure 4, it therefore appears that the extended wave system consists of repeated wave patterns, which begin with a characteristic emanating at the point (∞) on one blade and end with a characteristic emanating from the point (∞) on the adjacent blade. The nature of these wave patterns is such that no net effect is produced on the flow since the characteristics emanating from the points (∞) mark the beginning and the end of success-ive wave patterns, the undisturbed Mach number must exist in both direction and magnitude along these characteristics. The characteristics emanating from the points (∞) on the suction surfaces form parallel boundaries for the wave patterns, and extend upstream to infinity as Mach number. Downstream of the point (∞) the flow continues to expand (See Figure 4) so that it enters the blade row in an over-expanded state. The area normal to the streamlines must increase as the Mach number increases (supersonic) to satisfy continuity. Due to the small change of area which is necessary to satisfy this condition in the Mach number range under consideration (approximately ($1 \leq M_\infty < 1.7$), it is difficult to see the necessary area change from a geometric construction alone.

The position of the point (∞) on the blade where the Mach number and direction of the flow return to the relative undisturbed condition can be located in terms of the angle (θ_∞) by using a Prandtl-Meyer table and the appropriate chart. This follows, as by the Prandtl-Meyer relationships,

$$\theta_{\infty} = \theta_e + \nu_{\infty} - \nu_e$$

where

$$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}} \, \text{Tan}^{-1} \sqrt{\frac{\gamma-1}{\gamma+1}} \sqrt{M^2-1} \; - \text{Tan}^{-1} \sqrt{M^2-1}$$

The values of (ν) corresponding to (M_{∞}) and (M_e) are available in tables. This gives the direction of the undisturbed relative Mach number direction immediately. Similarly, one can find the Mach number at the leading edge (if supersonic), as

$$\nu_L = \nu_e - \theta_e$$

Note that this gives the Mach number behind the shock. In case the Mach number at the leading edge is subsonic, then one can locate the point on the blade, defined by the angle, (θ^*), where the Mach number equals one, as

$$\theta^* = \theta_e - \nu_e$$

In addition to the above information one can use a table or graph defining the detachment of an oblique shock in terms of the Mach number ahead of the shock and the wedge angle to determine when detachment occurs in the present case. For the case considered here, the Mach number ahead of the shock could be taken as (M_e) (the Mach number of the flow along the first characteristic to hit the leading edge) and the wedge angle would be (θ_e).

Thus, using the charts and simple Prandtl-Meyer relationships, one can relate a wide range of geometric parameters to the resultant flow field, to find:

1. The incidence angle of the undisturbed relative flow on the suction surface, (θ_{∞}).

2. The point on the blade surface (∞) where the Mach number and direction of the flow are the same as the relative undisturbed conditions.

3. The point on the blade, (e), where the flow just enters
    the blade passage, marking the end of the entrance region.

4. Mach number of the flow along the blade surface in the
    entrance region.

5. Conditions when the shock is detached.

A typical application of the results, would be for a blade row, where a particular axial and relative Mach number are desired. To start a design one could prescribe (M_∞), (M_{axial}) and (β_t). With (M_∞) and the axial Mach number prescribed, and (β_t) given, (θ_∞) is known immediately as $(M_\infty) \sin (\beta_t + \theta_\infty) = M_{axial}$. Utilizing the appropriate chart for (β_t), the curve for (M_∞) defines the relationship of (S/R) to (θ_e) and (M_e). By satisfying the conditions of equation 2 one obtains the value of (S/R) which satisfies the desired conditions.

A more direct application to a blade row design would be a case where $(\beta_t, S/R, M_\infty)$ are prescribed. Then the direction of the relative, Mach number (hence the axial Mach number and mass flow) follows immediately from the appropriate chart and equation 2.

CONCLUSIONS

The results indicate that the flow through a blade row with subsonic axial flow and supersonic relative flow is uniquely defined by the geometry of the entrance region of the blade row.

The influence of various geometric parameters describing the entrance region can be seen from the charts of the calculated results. The following conclusions may be drawn, for convex suction surfaces, from a study of the charts.

1. The relative undisturbed Mach number always has a negative incidence angle on the suction surface, except at the limiting case of sonic axial flow where the relative Mach number is just tangent to the suction surface at the leading edge.

2. Decreasing the blade spacing reduces the amount of over-expansion of the flow prior to entering the blade passage.

3. Increasing the radius of curvature in the entrance region has the same effect as decreasing the blade spacing.

4. Small values of (β_t) (implies low stagger angles) increases the amount of over-expansion of the flow prior to entering the blade passage.

REFERENCE

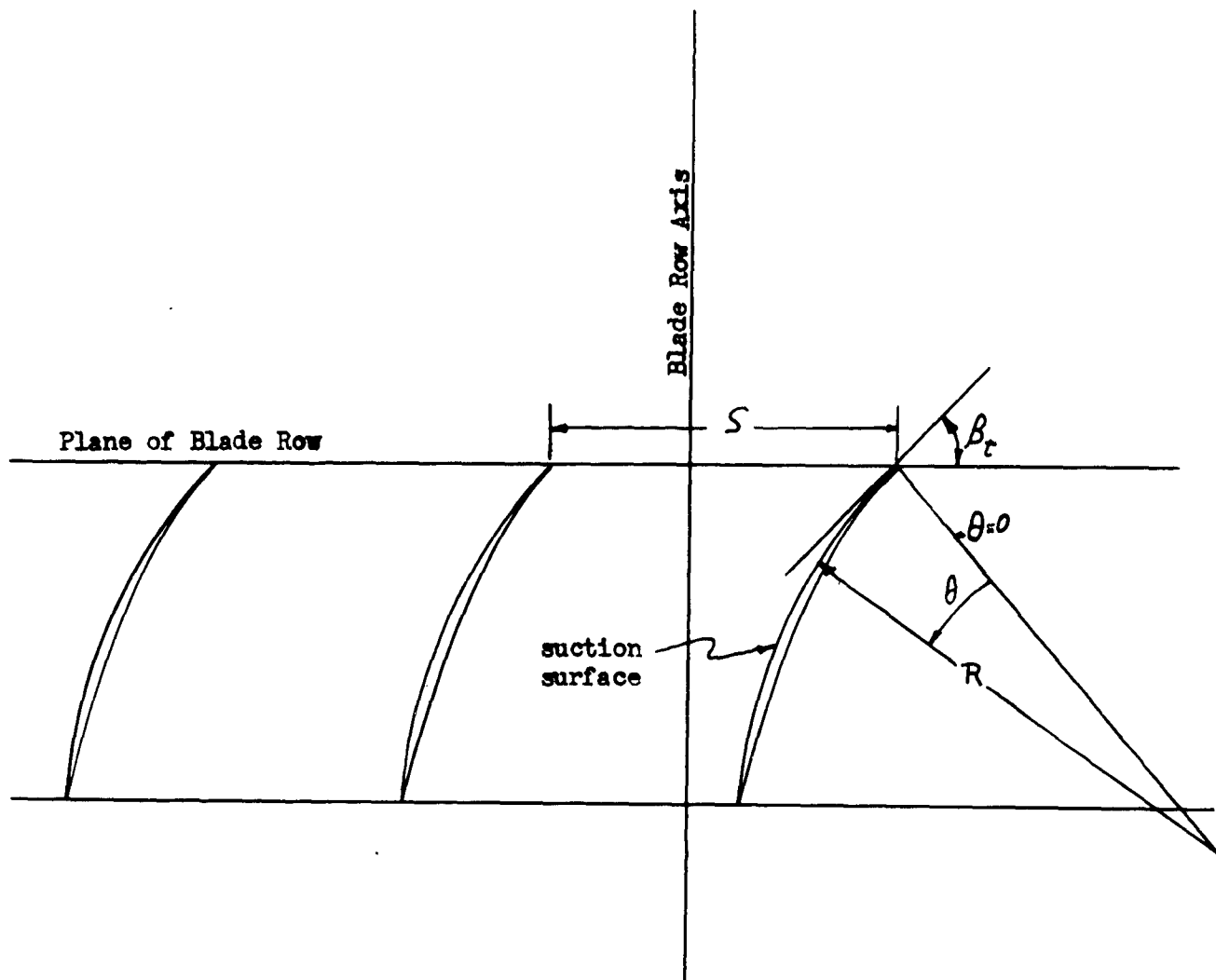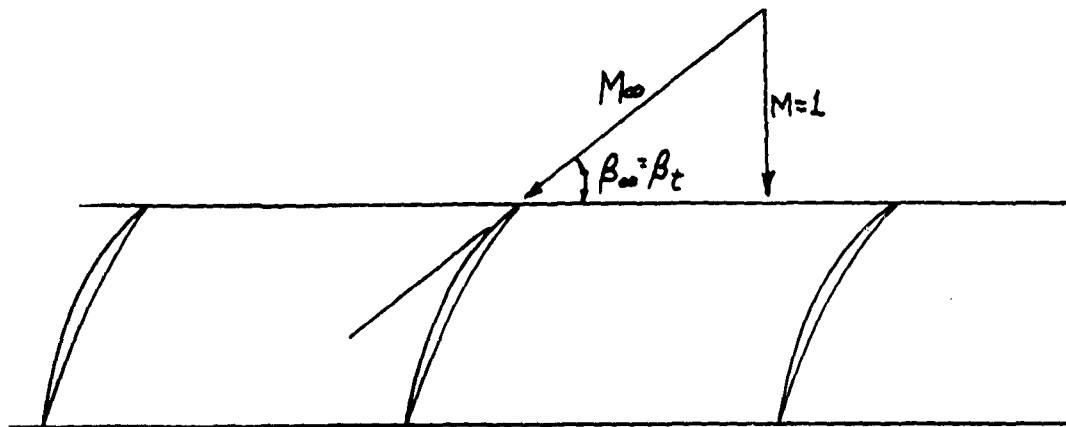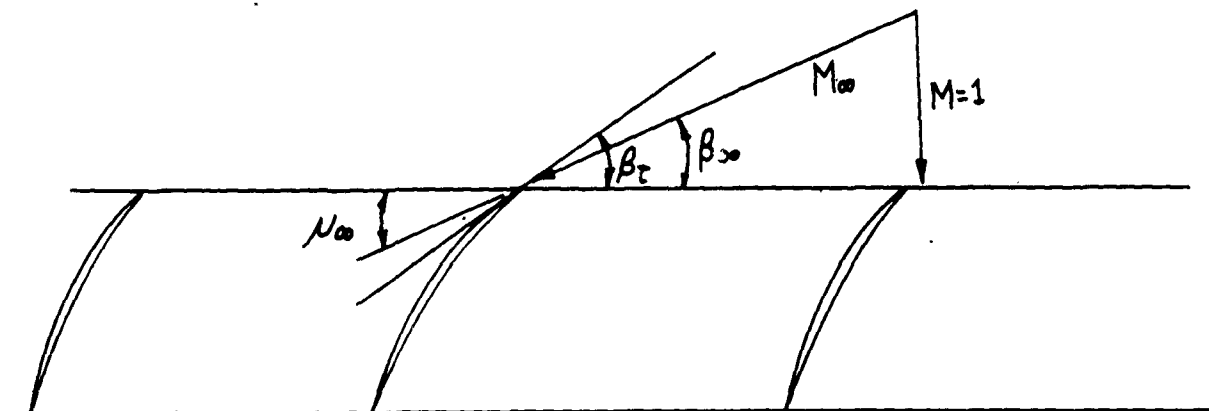1. Kantrowitz, A. ; "The Supersonic Axial-Flow Compressor" NACA Report 974, 1950.
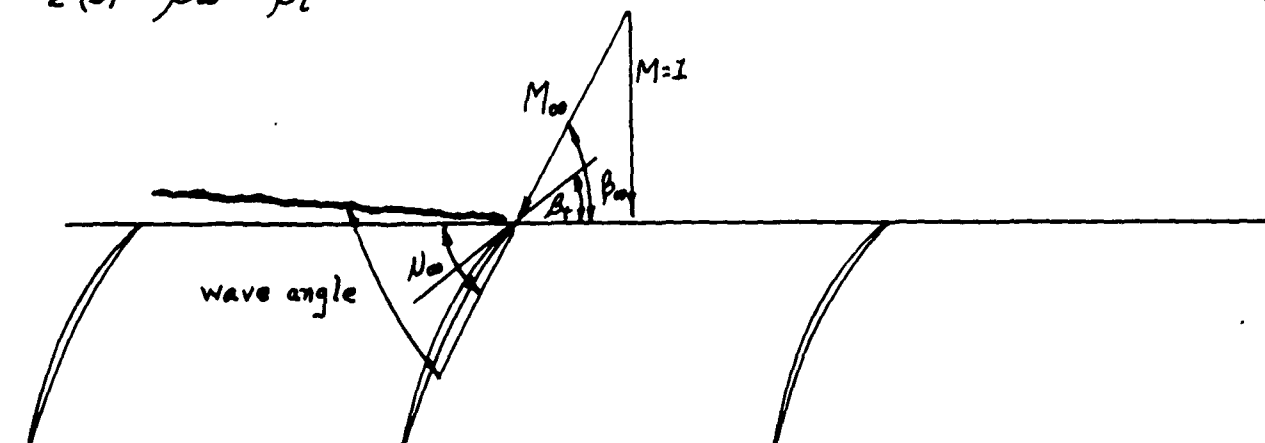
Figure 1. - Blade Row Geometry and Associated Notation

2(a) $\beta_\infty = \beta_t$

2(b) $\beta_\infty < \beta_t$

2(c) $\beta_\infty > \beta_t$

Figure 2. - Conditions for Sonic Axial Flow into a Blade Row
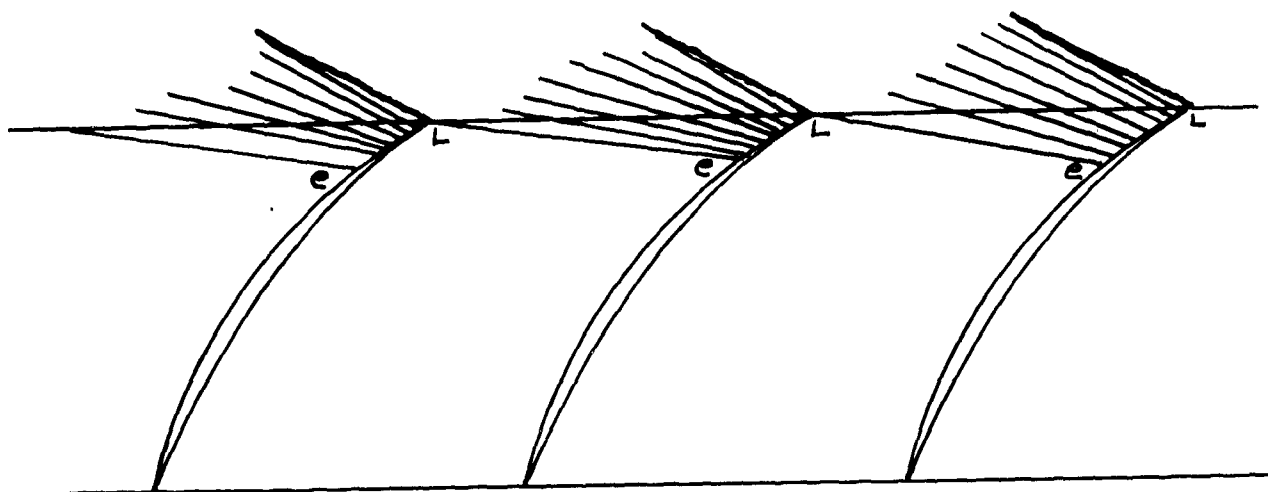
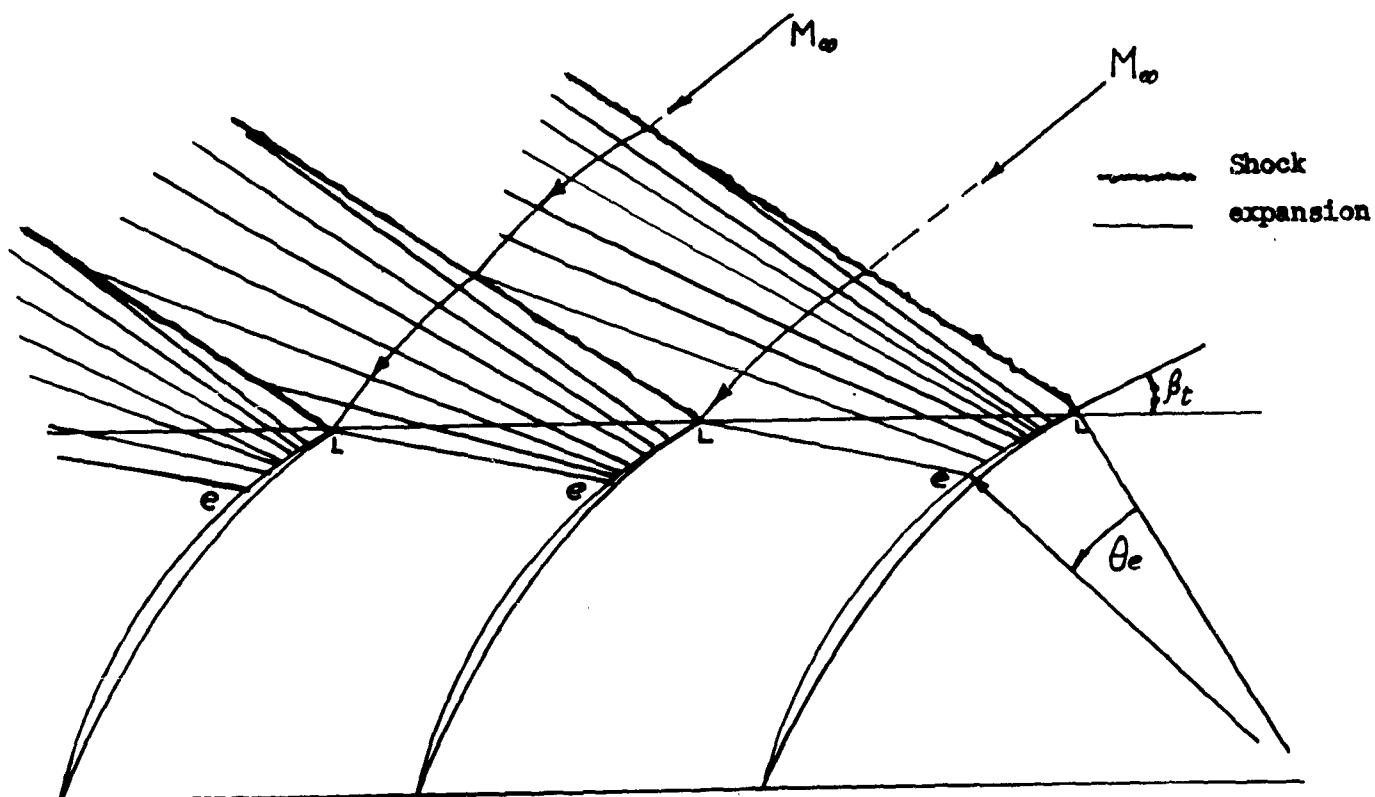Figure 3(a) - Schematic-Initial Step in Construction of Wave System

Figure 3(b) - Formation of the Steady Flow Wave System

Figure 3 - Construction of the Upstream Wave System for Subsonic
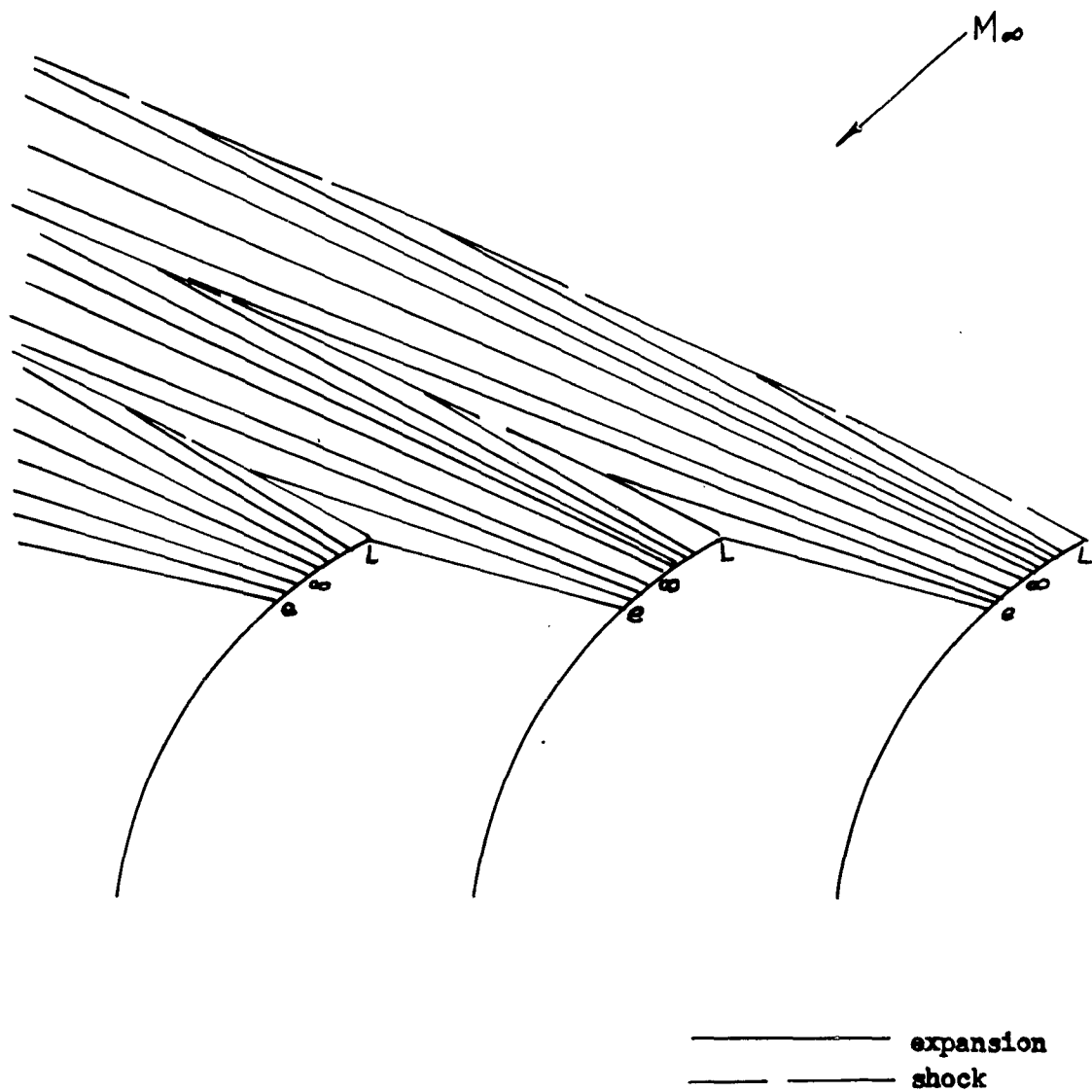Axial Flow into a Blade Row

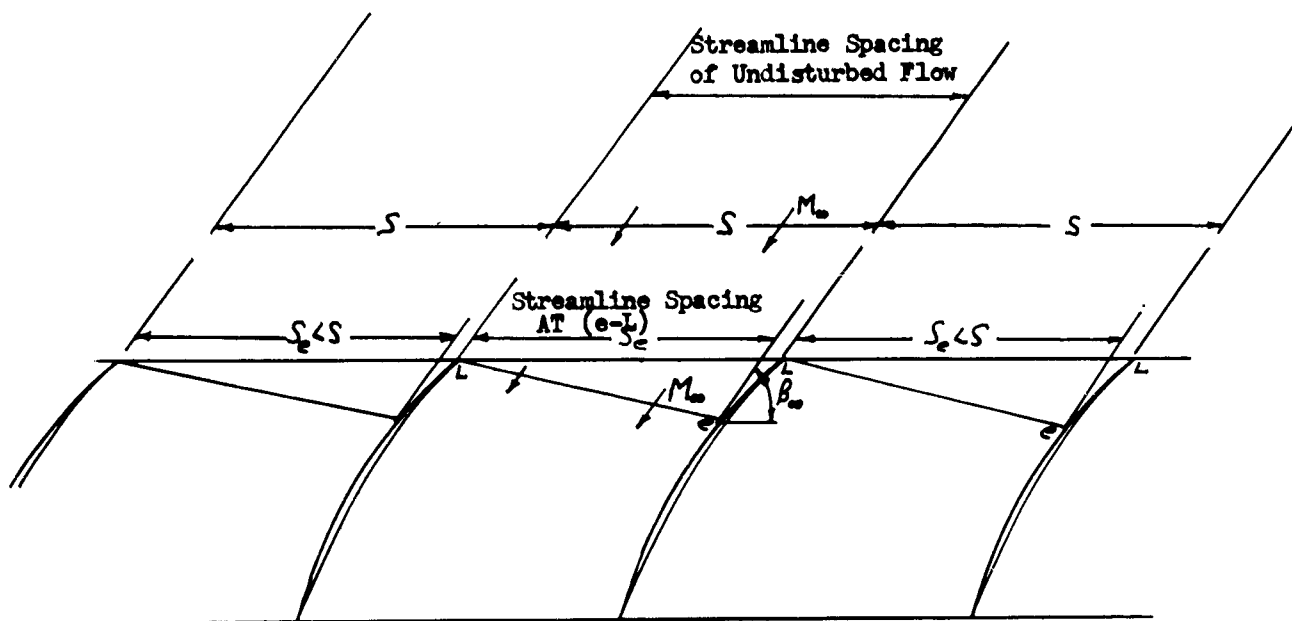Figure 4. - Steady Flow Wave System for Subsonic Axial Flow

Figure 5. – Geometric Construction for the Solution of the Flow
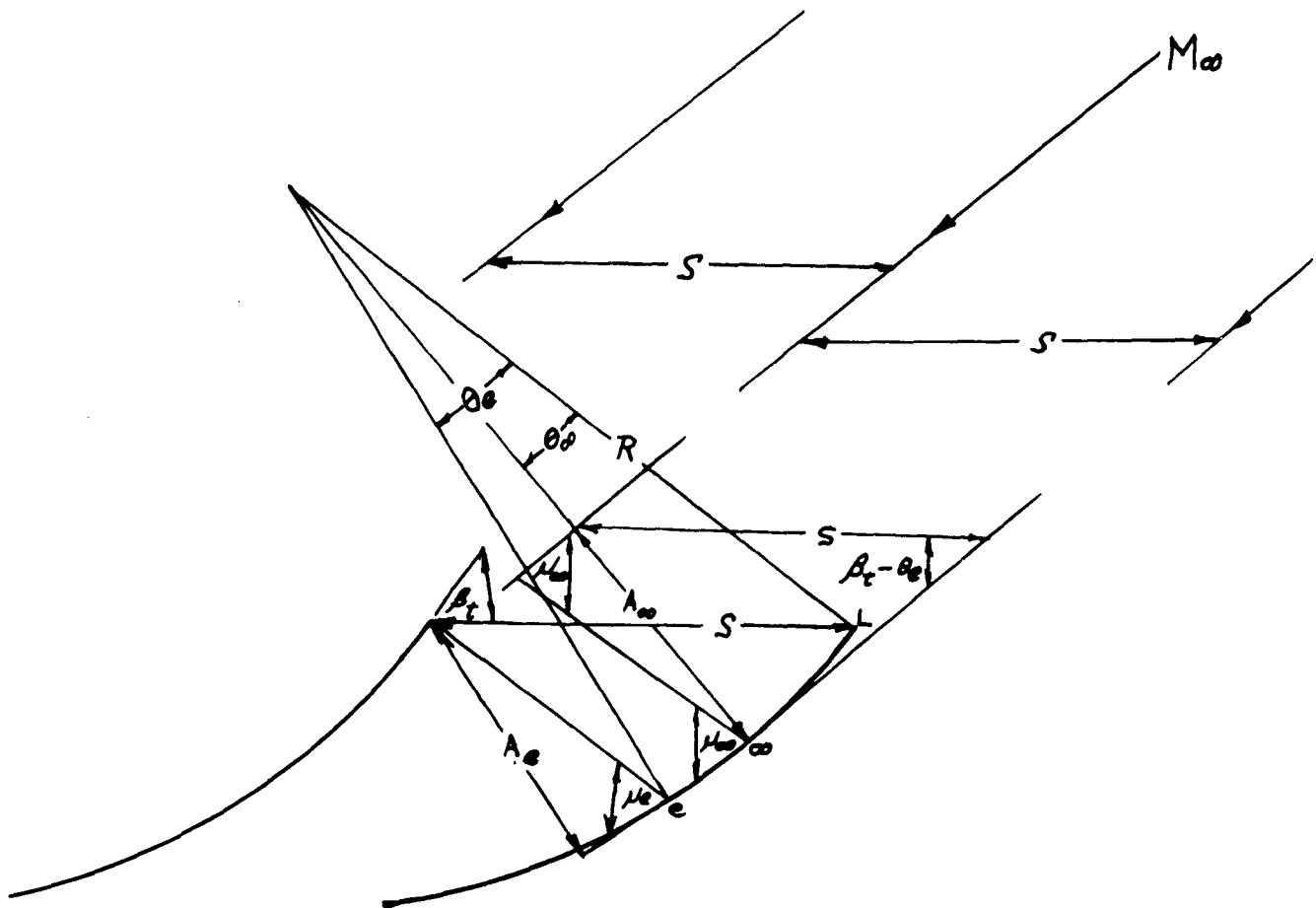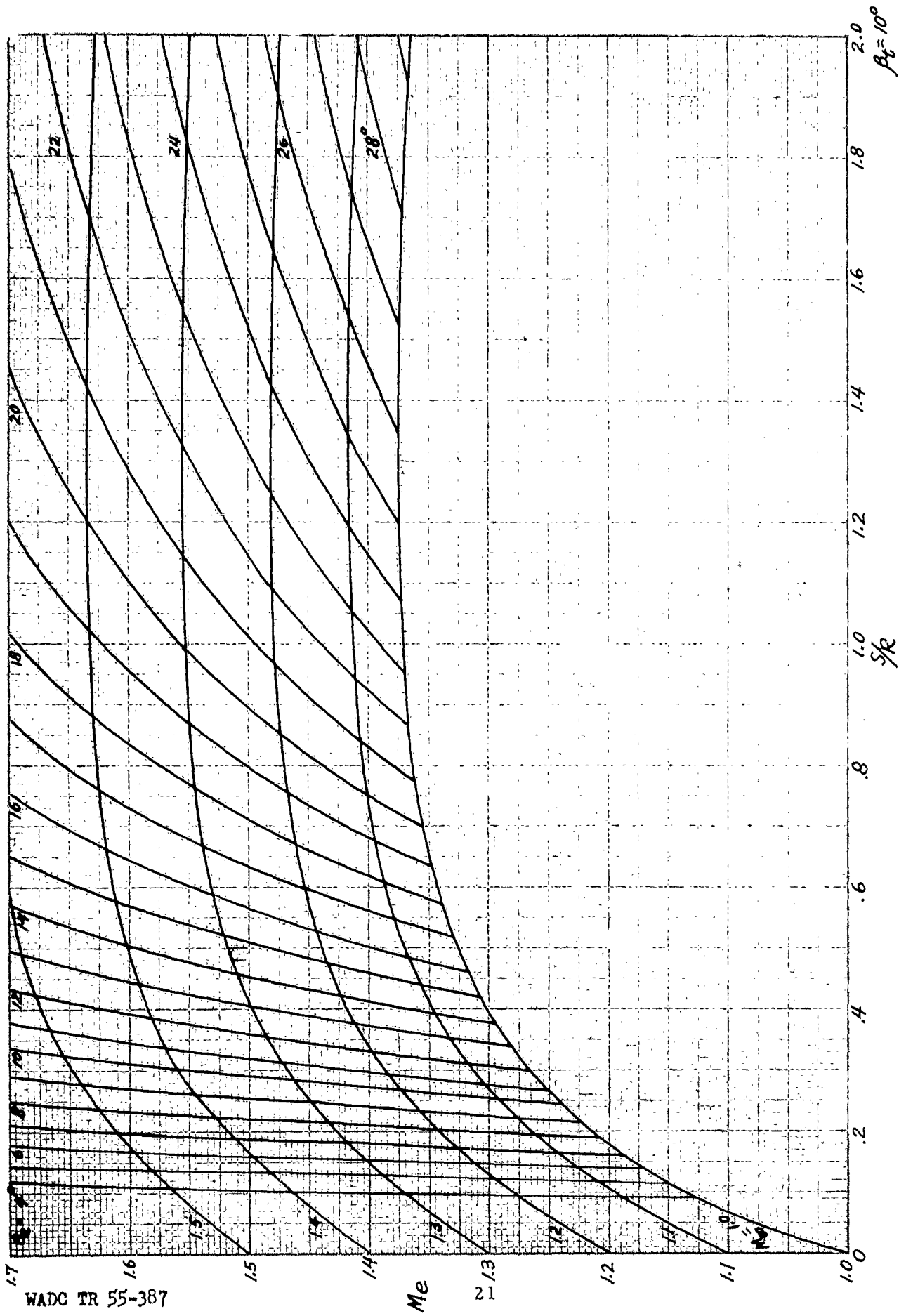Conditions–Continuity is not Satisfied

Figure 6. - Geometric Construction for the Solution of the Flow Conditions
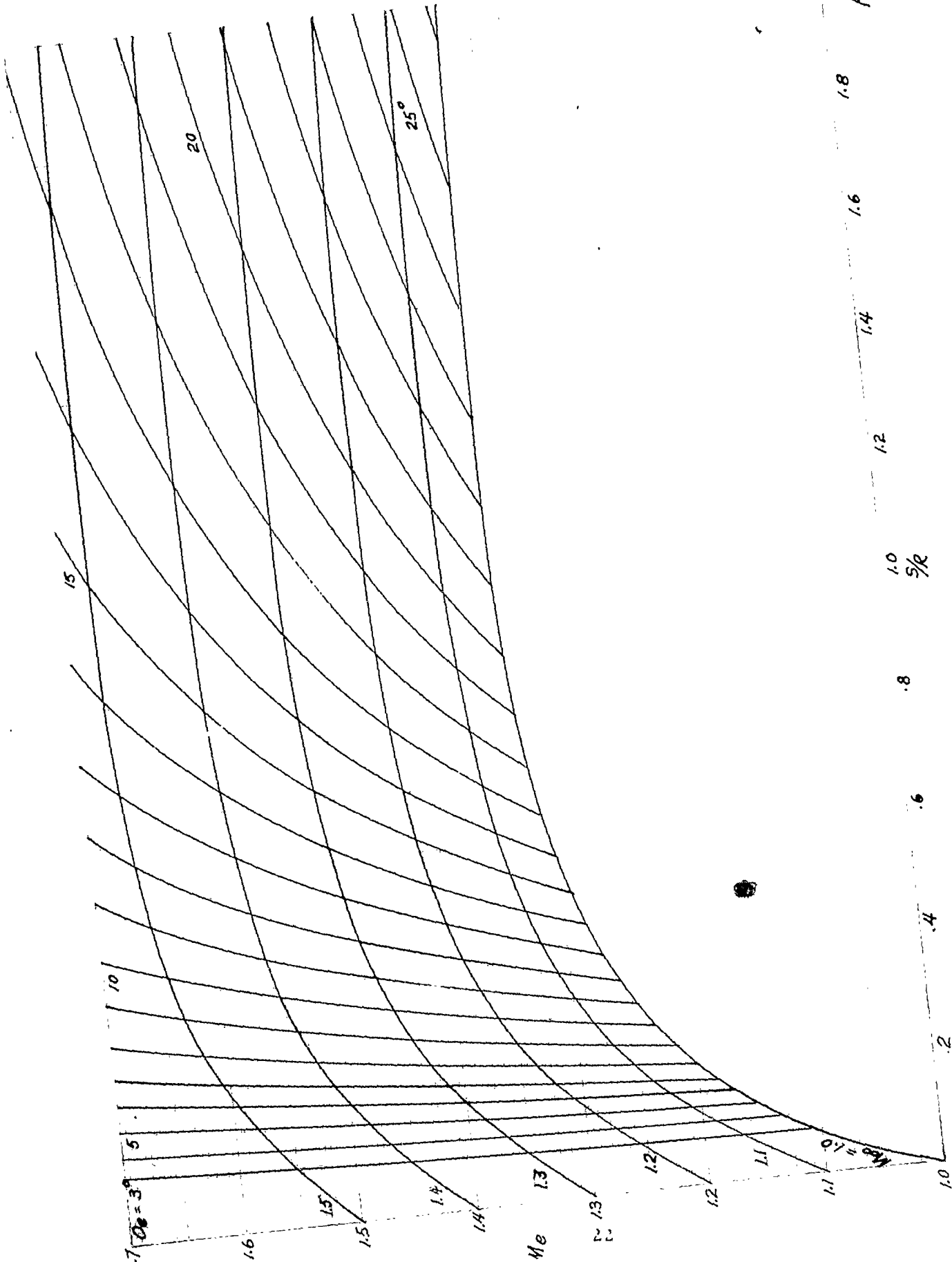in the Entrance Region for a Convex Suction Surface

Figure 7. - Geometric Construction for the Flow Conditions in the Entrance
Region for a Concave Suction Surface

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$\theta_e = 1°$

$\beta_t = 35°$

S/R

M_θ

26

$\beta_\ell = 40°$

$\theta_\ell = 1°$

5

$10°$

S/R

M_e

27